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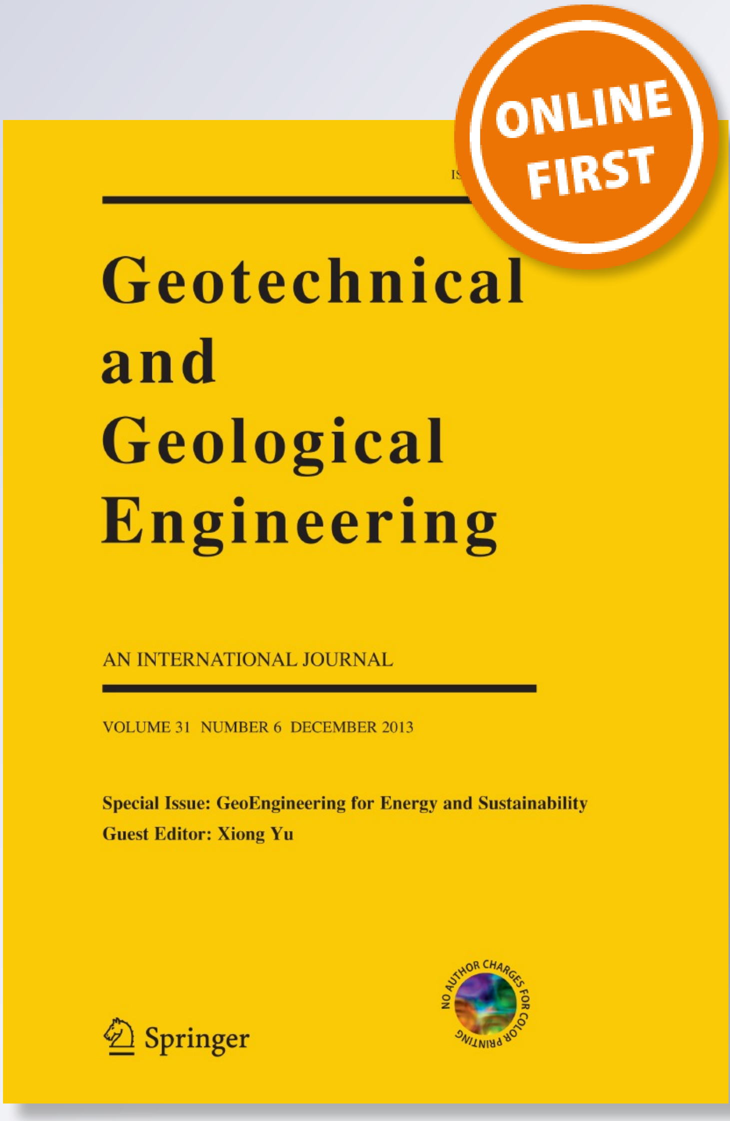

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ORIGINAL PAPER

Appropriate Uses and Practical Limitations of 2D Numerical Analysis of Tunnels and Tunnel Support Response

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Abstract In spite of the gradual development of three-dimensional analysis packages utilizing finite element models or finite difference algorithms for stress–strain calculations, two-dimensional (2D) analysis is still used as the primary engineering tool for practical analysis of tunnel behavior and tunnel support performance for design—particularly at the preliminary stage of a project. The applicability of 2D finite element analysis or analytical convergence confinement solutions to staged support installation depend on the application of an assumed or validated longitudinal displacement profile. Convergence in commonly applied 2D staged models is controlled by boundary displacement or internal pressure relaxation. While there have been developments to improve this methodology, this often assumes independence between the ground reaction curve and the support resistance, independence between the longitudinal displacement profile to support application, and the assumption that non-isotropic stresses and non-circular geometries can be handled in the same way as circular tunnels in isotropic conditions. This paper examines the validity of these assumptions and the error inherent in these extensions to 2D tunnel analysis. Anisotropic stresses and lagged (staged) excavation present a particular problem. Practical solutions are proposed for support longitudinal displacement LDPs in simplified conditions.

Keywords Weak rockmasses · Tunnel convergence · Linear displacement profile · Numerical modelling · Closure · Tunnel support

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1 Introduction

Tunnelling is an inherently three-dimensional (3D) process. The advancing tunnel face creates a complex 3D stress path as explored by Eberhardt (2001) and by Cantieni and Anagnostou (2009) among others. Tunnelling in yielding ground also generates a 3D, bullet-shaped zone of plasticity in soft rock. This developing plasticity or yielding zone, combined with the elastic closure of the surrounding rock mass creates a wall displacement profile (Fig. 1) that is non-linear, develops partially before the advancing face and continues for a number of tunnel diameters before equilibrium conditions are achieved. This profile, known as the longitudinal displacement profile (LDP) is a function of tunnel radius and the extent of the ultimate plastic radius (tunnel radius plus thickness of yielded ground). This relationship is explored in detail by Vlachopoulos and Diederichs (2009) for the axisymmetric condition (tunnel geometry and stress). This follows on from earlier work by Nguyen-Minh and

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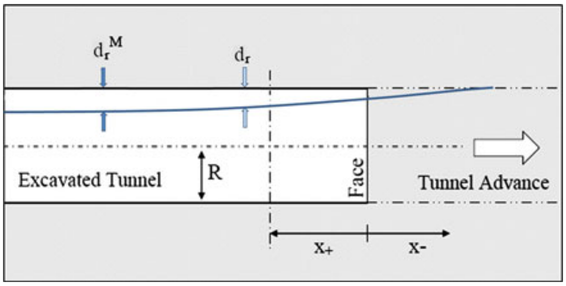

Fig. 1 Profile view of radial displacements (u_r) for an unsupported tunnel near the face

Guo (1996). Effect of tunnel support stiffness on ground response is explored in Bernaud and Rousset (1996).

In this paper, the authors examine issues related to two dimensional modeling, both as plane strain and as axisymmetric configurations and the relationships to true three-dimensional effects. In order to accurately simulate the loading of support or the effects of sequential excavation, the two dimensional(2D) model must capture the pre-face conditions (response in front of the tunnel face), the state of displacement and plasticity at the face and the subsequent development of deformation and yielding. It is important to examine these issues as 2D modeling is still very much state of practice for tunnel engineering analysis (Hoek et al. 2008).

The basic premise of 2D tunnel modeling is that the internal outward radial pressure applied to the tunnel boundary to replace in situ conditions reduces monotonically until the full excavation is represented (Cantieni and Anagnostou 2009). In cases where the in situ stress ratio is close to unity, this implies that the tunnel boundary moves (normally inwards) progressively as the tunnel face passes the modeled section. Ultimately, a stable tunnel closure is reached (for elasto-plastic analysis without strain softening and without ground surface interaction). This inward displacement of the tunnel boundary can be simulated by replacing the "rock" inside the tunnel with an outward pressure p_i (initially equivalent to the in situ pressure p_o) and reducing this internal pressure to zero over a number of model steps as in Fig. 2.

This reduction of the internal "support pressure" results in a redistribution of stress within the model and can lead to yielding of the rockmass around the tunnel. A plastic zone initiates in front of the tunnel or

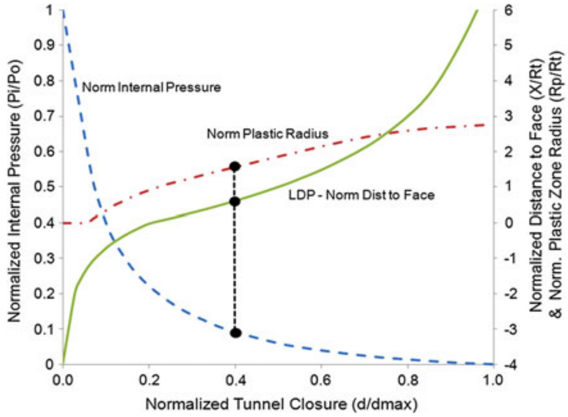

Fig. 2 Relationship between internal pressure, plastic radius, tunnel closure and position

sometime after the passage of the tunnel face and grows to a maximum, coincident with the maximum tunnel closure. Internal pressure and plastic zone radius are linked to tunnel closure (radial displacement). Closure, in turn, is linked to the actual axial position in the tunnel, relative to the tunnel face, through the longitudinal displacement profile or LDP.

Vlachopoulos and Diederichs (2009) made significant improvements to the calculation of the LDP for tunnels with extensive plastic zone development. This approach adjusts the longitudinal displacement profile for unsupported tunnels in order to account for plastic yield in front of the advancing tunnel and for the influence of excessive plastic radius. These original developments were directly intended for unsupported, full face circular tunnels in isotropic stress conditions with the associated practical limitations therein. As described in the next section, this technique is readily extrapolated to a staged excavation sequences, mildly anisotropic stress states and supported tunnels in practice, although there are limitations and caveats associated with this approach that will be discussed.

2 Model Approaches

A number of modeling approaches are discussed in this section. The following is a brief discussion of the model construction and execution. For the purposes of this investigation, Phase2 (Rocscience Inc. 2004) was used for the 2D numerical analysis and FLAC3D (Itasca 2005) was used for the 3D numerical analysis.

Phase2, a common, commercially available 2D numerical analysis software package, utilizes the implicit finite element method (FEM) (i.e. solves the mathematical relations) while FLAC3D employs the finite difference method (FDM) (i.e. solves the physics of the problem) in its determinations. Both of these program are widely used in the rock mechanics community for design purposes as well as to capture the behaviour of a tunnel (i.e. stress re-distributions and displacements) associated with tunnel excavation. Cai (2008) has investigated the numerical modeling codes for Phase2 and FLAC (2D) (the basis of FLAC3D) on the influence of stress path on tunnel excavation response and these findings will not be repeated here. Chai stated that one software package was not superior to the other, rather he points out the importance of understanding the program codes and selecting the right tool and modeling approach to represent the expected stress path as close to reality as possible. The emphasis in comparison therefore, should not lie in the limitations of the software packages but on the details of how the true physical phenomenon is being modeled.

2.1 3D Finite Difference Plasticity Model

FLAC3D (Itasca 2002) is an explicit finite difference program that is used to study the mechanical behaviour of a continuous three-dimensional medium as it reaches equilibrium or steady plastic flow. The response observed is derived from a particular mathematical model and from a specific numerical implementation. The mechanics of the medium are derived from general principles (definition of strain, laws of motion), and the use of constitutive equations defining the material. The resulting mathematical expression is a set of partial differential equations, relating mechanical (stress) and kinematic (strain rate, velocity) variables, which are to be solved for particular geometries and properties, given specific boundary and initial conditions. It is the inertial terms that are used as means to reach (in a numerically stable fashion) the equilibrium state. The solid body is divided into a finite difference mesh of 3D zones. Within a cycle, new velocities and displacements are determined from forces and stresses using the equations of motion. Strain rates are determined from velocities and new stresses from the strain rates.

The advantage of using the explicit finite difference formulation is that the numerical scheme stays stable even when the physical system may be unstable. This is particularly advantageous, when modeling non-linear, large strain behavior and actual instability. FLAC3D conducts a check of the element state at each time step with respect to the yield criterion. However, the disadvantage of the time-marching explicit scheme of the FLAC3D is that calculation times can be longer than those of implicit formulations although memory requirements are reduced as explicit methods that do not have to store matrices for calculations of equilibrium.

The 3D models that were developed for the purposes of this investigation can be seen in Fig. 3. These models consisted of circular and horseshoe excavation geometries incorporating sequential excavation and support. The numerical models that were created use FLAC3D group zones. The models were 110 m in height and 110 m wide with a tunnel length of 100 m (depth). (Figure 3a is cut away to show tunnel.) The excavated material within the tunnel was created separately and was subdivided into subsections that constituted an excavation step and could be separated into full-face or top heading/bench excavation. At each excavation step, an excavation sub-section block was 'nulled' and steps were conducted in order to ensure equilibrium conditions were met prior to the next excavation sequence. The tunnel lining consisted of a 30 cm thick shotcrete that was replicated using liner elements.

2.2 Phase2:Plane Strain 2D Numerical Modeling

While 2D analysis are also possible with the finite difference code FLAC, a finite element approach is used here for comparison. The two 2D methods give comparable results for elasto-plastic problems. Phase2 is a 2D, implicit, elasto-plastic finite element method program based on the finite element formulation and the strain-softening/hardening formulations described in Owen and Hinton (1980) and Chen (1982), respectively. The load stepping and iterative plastic solution described by these authors is used here. The 2D models that were developed for the purposes of this investigation can be seen in Fig. 4. Both fixed displacement and constant pressure boundary conditions are used in this analysis.

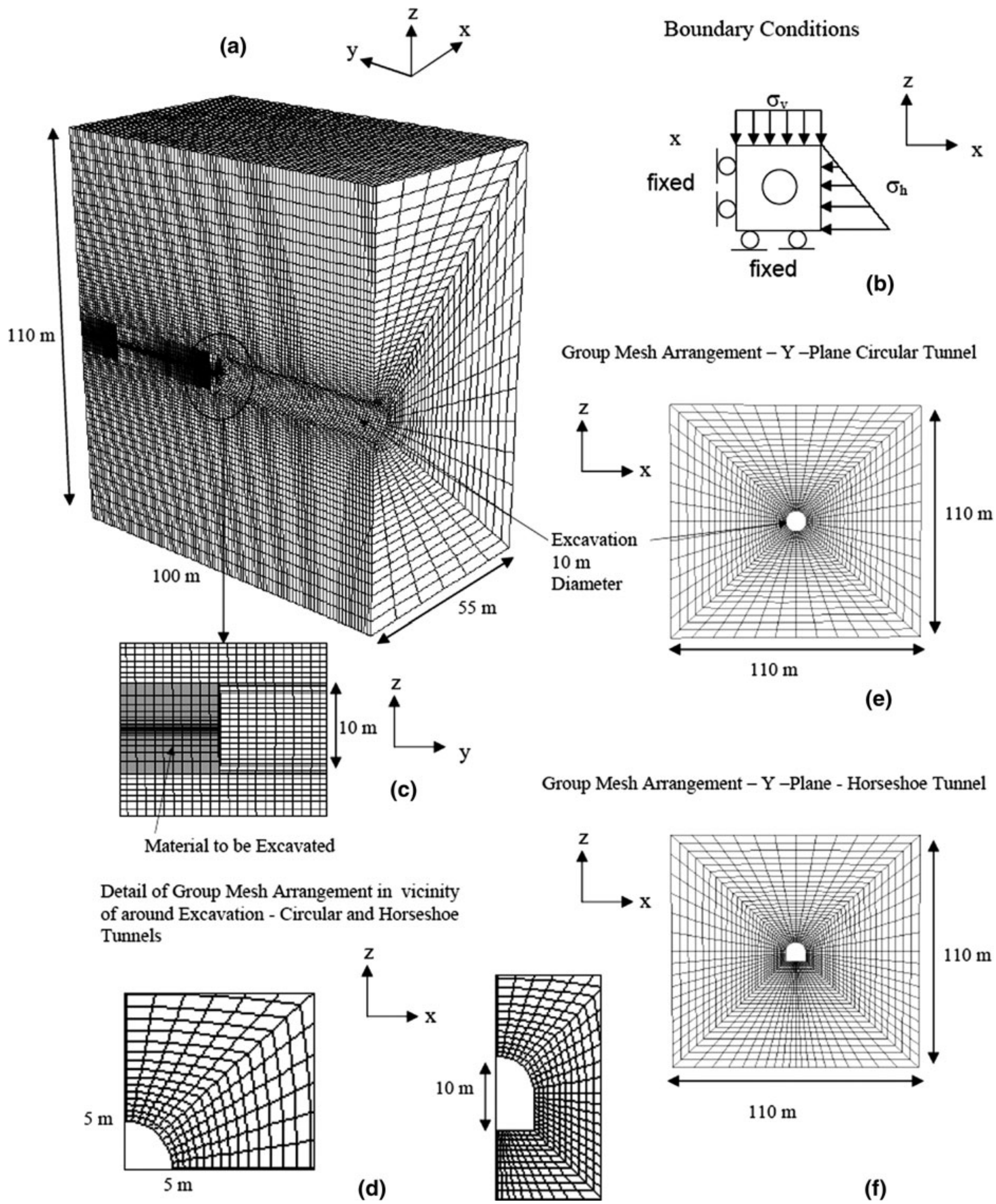

Fig. 3 Geometry and Boundary Conditions of the FLAC3D Finite difference model: **a** Cutaway of model to show tunnel (not a half-symmetry model); **b** boundary conditions;**c** mesh for *circular tunnel*; **e** and **f** mesh detail for *horseshoe tunnel*

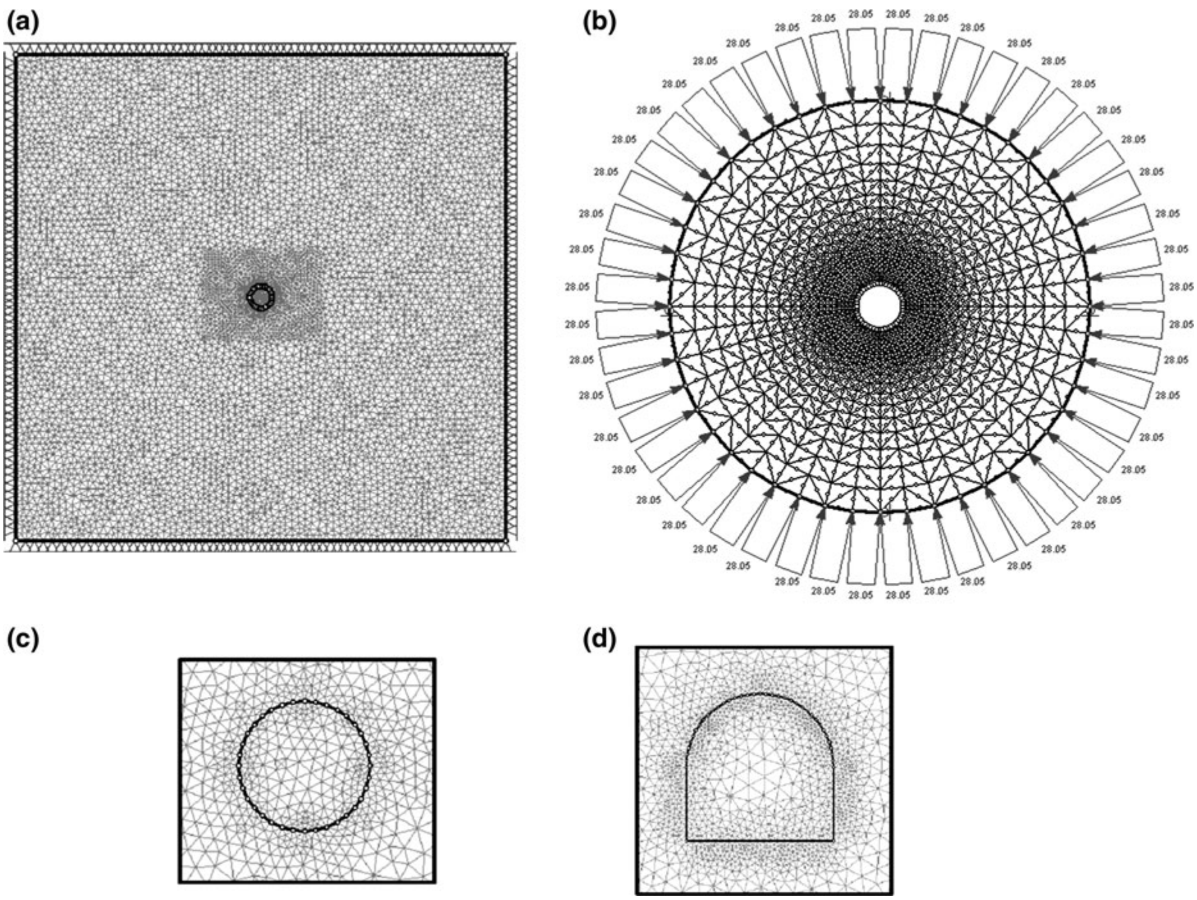

Fig. 4 **a** Phase2 model with fixed boundary conditions some distance (normally expressed as a multiple of tunnel radii) from the tunnel. **b** Phase2 model with pressure boundary conditions. Detail of **c** *circular* and **d** *horseshoe tunnel* geometries

2.3 Phase2:Axisymmetric 2D Numerical Modeling

Phase2 also allows computation in axisymmetric mode. In this formulation, a 2D model geometry is used with a single axis of symmetry about which the model is assumed to be 3D through geometric rotation. Also in this formulation, the equivalent 2D equilibrium and strain–displacement relationships (with respect to deformation in the model plane) are a function of radial distance from the axis of symmetry (see Brady and Brown 1993, for example). In these analyses, the tunnel is advanced in steps from the bottom up in a model that is centered on the tunnel axis as shown in Fig. 5. The excavation is conducted incrementally by removing the tunnel material. Vlachopoulos and Diederichs (2009) showed that a tunnel advance increment below 0.4D was sufficient to simulate continuous excavation without significant practical error. These models advance at a rate of 0.2D or 2 m for a 10 m tunnel (5 m radius).

2.4 Review of Selected Methods of Analysis for Capturing 3D Effects in 2D

As the effect of an excavation in a rock mass is clearly a 3D process, the ensuing deformations cannot be simulated directly in 2D finite element plane strain analysis. 2D axisymmetric modeling does practically reproduce 3D effects for very simple cases (circular geometry and isotropic material and stress) with moderate degrees of squeezing and limited non-linear behaviour. In 2D plane strain, the progressive displacement of the tunnel boundary must be recreated in accordance with the appropriate linear displacement profile. If done correctly, this will capture the

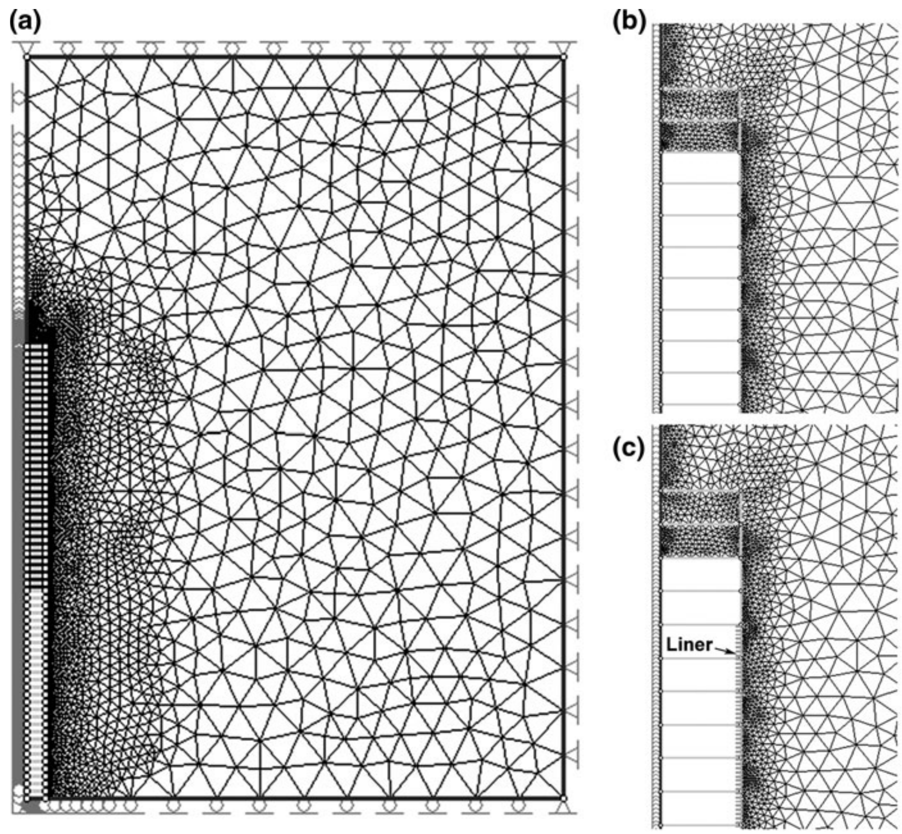

Fig. 5 **a** Axisymmetric PHASE2 model with fixed boundary conditions. Axis of Symm. is on righthand vertical edge. **b** Detail of stepwise excavation (tunnel advance is from *bottom to top*). **c** a liner is installed 2 rounds back from the face

progressive development of loads and displacements in tunnel geometries and in support elements that respond in the radial plane (liners and bolts for example, but no forepoles and face support). The LDP is recreated implicitly in 2D plane strain. According to a number of methodologies such as those described by Karakus (2006). The methodologies commonly employed in current design practice for 2D modeling to mimic real 3D effects are (Fig. 6):

a) Straight excavation,
b) Field stress vector/average pressure reduction,
c) Excavation of concentric rings, and
d) Face de-stressing (with or without softening).

2.4.1 Straight Excavation

This method (Fig. 6a) is simply the full-face excavation usually associated with hard rockmasses of a homogeneous nature and with simple tunnel geometries. Initially, the material properties of the geomaterial as well as the simplistic tunnel shape are input. In the next sequence of 2D numerical modeling, the material is excavated in its entirety allowing instantaneous displacements to be determined without giving any other consideration to possible influencing mechanisms (i.e. support provided by the face, 3D plasticity effects in front of the face, capturing of pre-convergence etc.). This is a simplistic excavation technique that does not take into account the stress redistribution and ensuing deformations that occur during the sequential excavation and advance of the tunnel that can be more effectively simulated in 3D numerical models.

2.4.2 Average Pressure Reduction (convergence-confinement method)

Convergence-confinement analysis or the stress relief method (Panet 1993, 1995; Carranza-Torres and Fairhurst 2000; Duncan-Fama 1993 and others) is a widely used tool for preliminary assessment of squeezing potential and support requirements for circular tunnels in a variety of stress states and

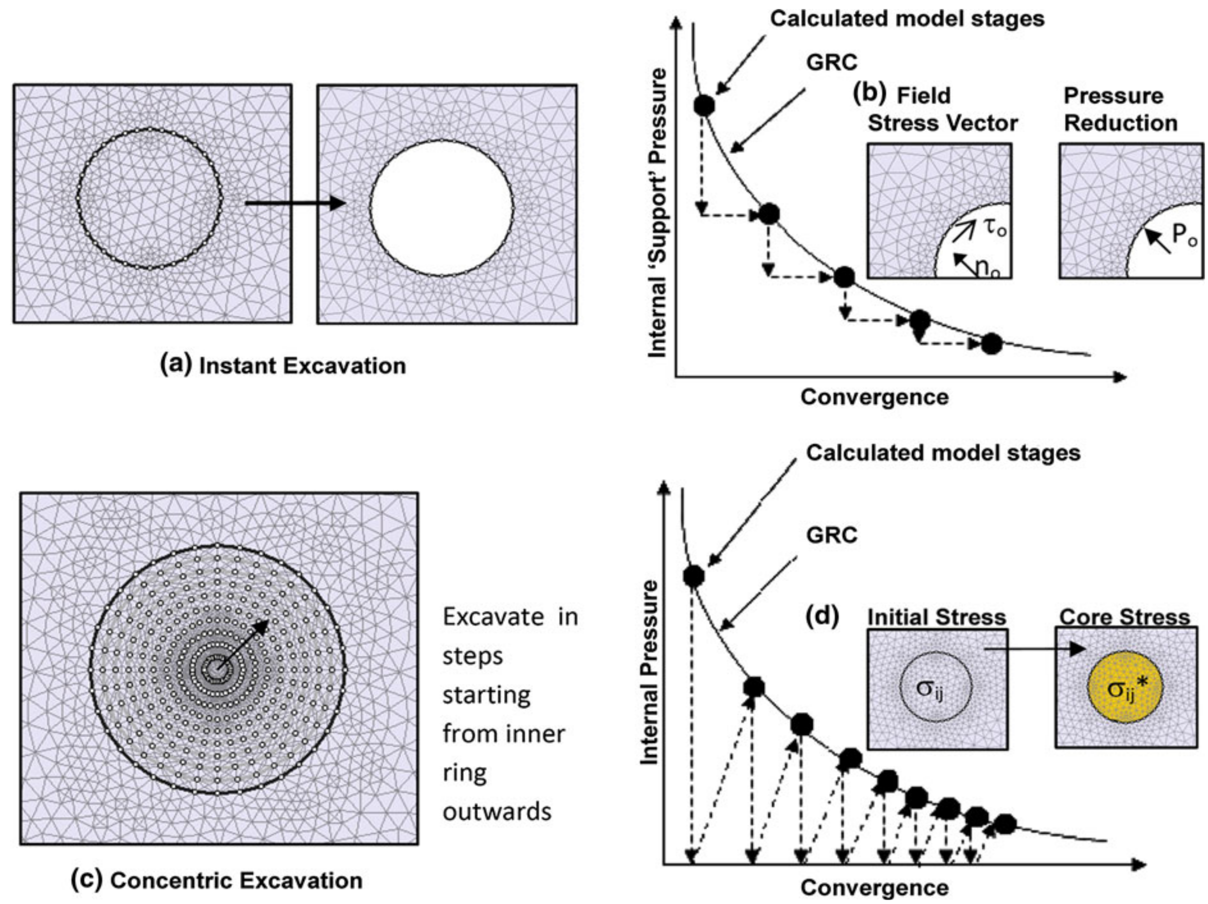

Fig. 6 Methods or advanced strategies used in 2D numerical analysis in order to approximate the uniquely 3D behaviour associated with rock tunnel excavation

geological conditions. An internal pressure (P_o), initially equal to the in situ stress is applied on the inside of the excavation boundary. The pressure is incrementally relaxed until the excavation boundary condition is effectively zero normal stress. The extent of plastic yielding and thereby, the boundary deformation is calculated at each stage of the process. The result is a continuous representation of the deformation-internal pressure relationship for the tunnel given a particular material strength, deformability, dilation and stress state. The internal pressure is not a direct representation of real effects, however, it is a substitute for the effect of the gradual reduction of the resistance due to the effect of the distancing supporting tunnel face. The internal pressure that is coupled with a given boundary displacement is a measure of the amount of support resistance required to prevent further displacement at that point in the progressive tunneling model. The stress is applied normal to the inner boundary and idealizes the progressive stress state. Also referred to as the load step method, as there is an incremental reduction of tunnel boundary tractions that simulate advance.

2.4.3 Field Stress Vector

In cases where the initial stresses are not isotropic, the boundary pressure in the convergence-confinement technique must be replaced with a traction vector with shear (τ_o) and normal (n_o) components applied to each tunnel boundary element to replace the in situ stress acting on the element plane pre-tunnel. In this technique, in terms of the Ground Reaction Curve(GRC) (convergence vs internal support pressure), there is an incremental reduction (dashed line) of tunnel boundary tractions that simulate progressive tunnel excavation advance. (Figure 6b).

2.4.4 Concentric Disks of Excavation Method

This is a method that excavates the tunnel cavity in stages concentrically from the centre of the tunnel to the outer boundaries of the desired tunnel diameter (or shape). This can be seen in Fig. 6c for a circular tunnel. Each excavation disk that is nulled in this system of excavations represents a different stage of tunnel advancement. In a 3D sense, the excavation of the central disk represents a weakening of the material ahead of the excavated face while the final ring that is excavated represents the open cavity and passing of the face past that location. This method can also be combined with softening or distressing of the material, whereby one would reduce the Modulus of Elasticity (E_i) of the core material from its original value, E. This method is still used in practice but will not be discussed further here.

2.4.5 Face Replacement or Destressing

Plane strain simulation of tunnel advance in this method involves the replacement of the tunnel core with unstressed, elastic material during each step. The tunnel boundary is allowed to converge during the subsequent model step until the stresses reestablish in the tunnel core and a temporary equilibrium is reached. The face is then replaced again and this process is repeated. In this way, the tunnel works its way down the pressure–displacement (ground-reaction) curve in a series of steps. This method (Fig. 6d) was favoured in the past as the stress-vector technique was difficult to incorporate manually into a model. These two methods will be compared here.

The face replacement method can be made more efficient by progressive softening of each successive core replacement. Softening the face on its own will not create a response as the model functions on the basis of stress equilibrium (resetting the stiffness does not create a force imbalance in the model and therefore

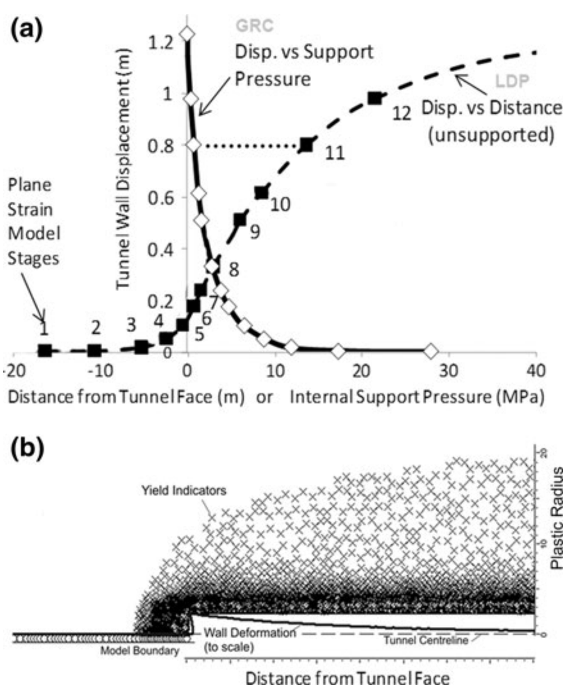

Fig. 7 **a** Ground reaction curve, "Disp. versus Support Pressure" and corresponding longitudinal displacement profile "Disp versus Distance (unsupported)". Normalized plastic radius Rp/Rt = 8 in this example. Point symbols and number ID's represent corresponding stages in plane strain model (related symbols are linked horizontally between two curves as shown for stage 11 by dotted line). **b** Axisymmetric model equivalent to the 2D plane strain analysis in (a). Note that Rp is the radius of the plastic zone and Rt is the radius of the tunnel

Table 1 Parameters used for 2D and 3D model comparisons

Parameter	Material			
	B	C	D	E
p_o/σ_{crm}	8	6	4	2
σ_{ci} (MPa)	35	35	50	75
mi	7	7	7	7
ν	0.25	0.25	0.25	0.25
γ (MN/m^3)	0.026	0.026	0.026	0.026
E_i	19,212	19,249	27,630	21,567
po (MPa)	28	28	28	28
GSI	35	45	48	60
m[a]	0.687	0.982	1.093	1.678
s[a]	0.0007	0.0022	0.0031	0.0117
a[a]	0.516	0.508	0.507	0.503
E_{rm} (MPa)	2,183	4,305	7,500	11,215
σ_{crm} (MPa)[a]	3.5	4.7	7	14
c (MPa)[a]	1.100	1.753	2.145	3.259
φ	21.50	23.71	27.05	33.40

[a] Values calculated based on Hoek et al. 2002

no direct response). Softening combined with face replacement (or distressing) results in an efficient excavation sequence simulation.

2.5 Applying the Longitudinal Displacement Profile

The LDP is one of the three basic components of the convergence-confinement method. A characteristic LDP diagram indicates that for a tunnel of radius R_t there is an amount of axial displacement (d) at some distance (−X) ahead of the face (i.e. a zone of influence prior to excavation of the core beyond the face) and at certain distance behind the face (+X) that the amount of displacement approaches a constant value d_{max} (Carranza-Torres and Fairhurst 2000). As shown by Vlachopoulos and Diederichs (2009) the normalized LDP (d/d_{max} vs X/R_t) is a function of the ultimate plastic radius in an alternative approach to that proposed by Corbetta et al. (1991).

The first step in the analysis process is to determine the maximum plastic radius via a simple plane strain

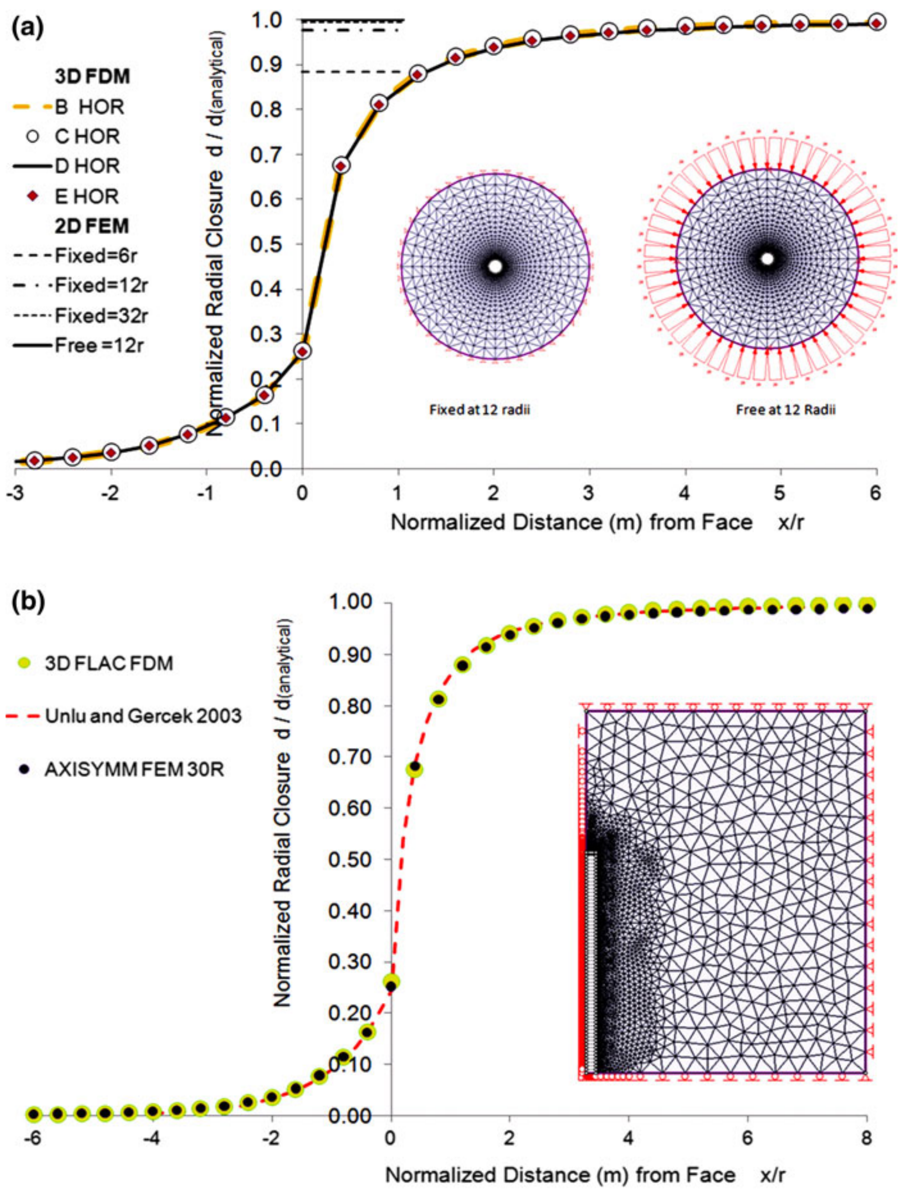

Fig. 8 **a** Elastic LDP results calculated using FLAC3D *circular tunnel* models (isotropic stresses and elastic properties as per Table 1. Comparison with 2D analytical solution for final displacement is shown. **b** Elastic LDP results calculated using FLAC3D *circular tunnel* models (isotropic stresses and elastic properties as per Table 1.) Comparison with 2D axisymmetric solution and analytical result

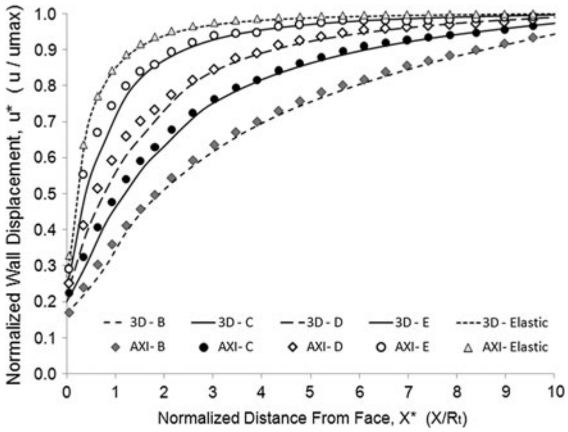

Fig. 9 Plastic LDP results calculated using FLAC3D *circular tunnel* models. Comparison with 2D axisymmetric solution

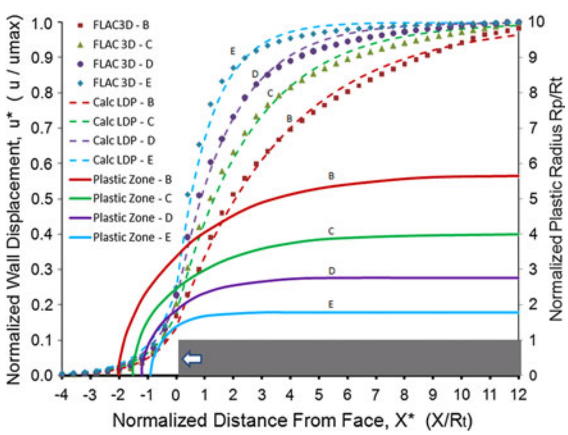

Fig. 10 Plastic LDP results calculated using FLAC3D *circular tunnel* models. Comparison with 2D analytical (calc) solution from Vlachopoulos and Diederichs 2009. Development of plastic radii in FLAC3D models is shown

analysis of the unsupported tunnel or through an analytical solution such as that given by Carranza-Torres and Fairhurst (2000). Next, the longitudinal deformation profile can be calculated using the methodology of Vlachopoulos and Diederichs (2009). Alternatively, an axisymmetric model can be used for this purpose, facilitated by the assumed isotropic stresses and circular profile. A longitudinal deformation profile for an unsupported tunnel is developed as shown by the solid line ("Disp. vs Location") in Fig. 7.

A 2D finite element plane strain analysis was then applied to the full face construction sequence (unsupported). The technique of progressive face replacement (distressing) described in the previous section

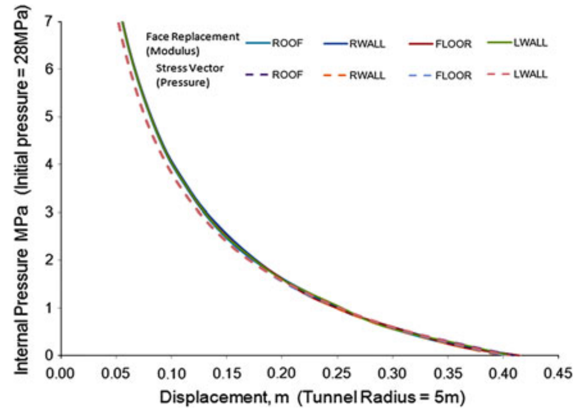

Fig. 11 Comparison of Ground Reaction Curve for idealized *circular tunnel* plane strain analysis

was applied. At the end of each stage in the 2D model, the tunnel wall will have moved a certain distance.

In addition, there will be a certain pressure or traction on the surface (applied by the reloaded core in this case or applied directly in the stress-vector approach). The incremental displacement–pressure value pairs collectively define the (GRC)(white diamonds on "Disp. vs Support Pressure" curve in Fig. 7a). Each stage can also be associated with a point on the LDP defining locations along the tunnel using the longitudinal deformation profile (shown for stage 11 by the dotted line). Model stages can be adjusted in "space" along the tunnel by adjusting the pressure increments or face replacement modulus values.

This methodology has been used in one form or another throughout the tunnelling industry. This technique works as intended for unsupported circular tunnels with isotropic stress fields - the same assumptions implicit in axisymmetric analysis (Fig. 7b).

Inaccuracies can be expected when the plane strain analysis includes anisotropic stresses, staged support, non-circular geometries and staged excavations. The rest of this paper will explore the significance of these inaccuracies and possible solutions.

3 Boundary Conditions and 2D Method Comparison

The comparisons that follow were conducted using supported and unsupported simulations with elastic and elastic-perfectly plastic models (Mohr–Coulomb constitutive model within FLAC3D and Phase2). The materials and input parameters were selected in order to span the spectrum of the ratio of rock mass strength to in situ stress and strain considerations. The suite is similar to that used in Vlachopoulos and Diederichs 2009. The parameters or properties associated with each material B, C, D and E are located in Table 1. As can be seen, materials B, C, D and E have a p_o/UCS_{RM} (in situ pressure to rockmass uniaxial compressive strength) ratios of 8, 6, 4 and 2 respectively. Mohr–Coulomb equivalent properties and rockmass strengths were estimated as per Hoek et al. (2002) and the elastic moduli were estimated based on Hoek and Diederichs (2006).

3.1 Boundary Conditions

It is important to establish the influence of 2D boundary conditions to ensure that valid comparisons can be made. Two options are explored here—fixed displacement (=0) outer boundary conditions some distance away from the excavation (as in Fig. 5) and

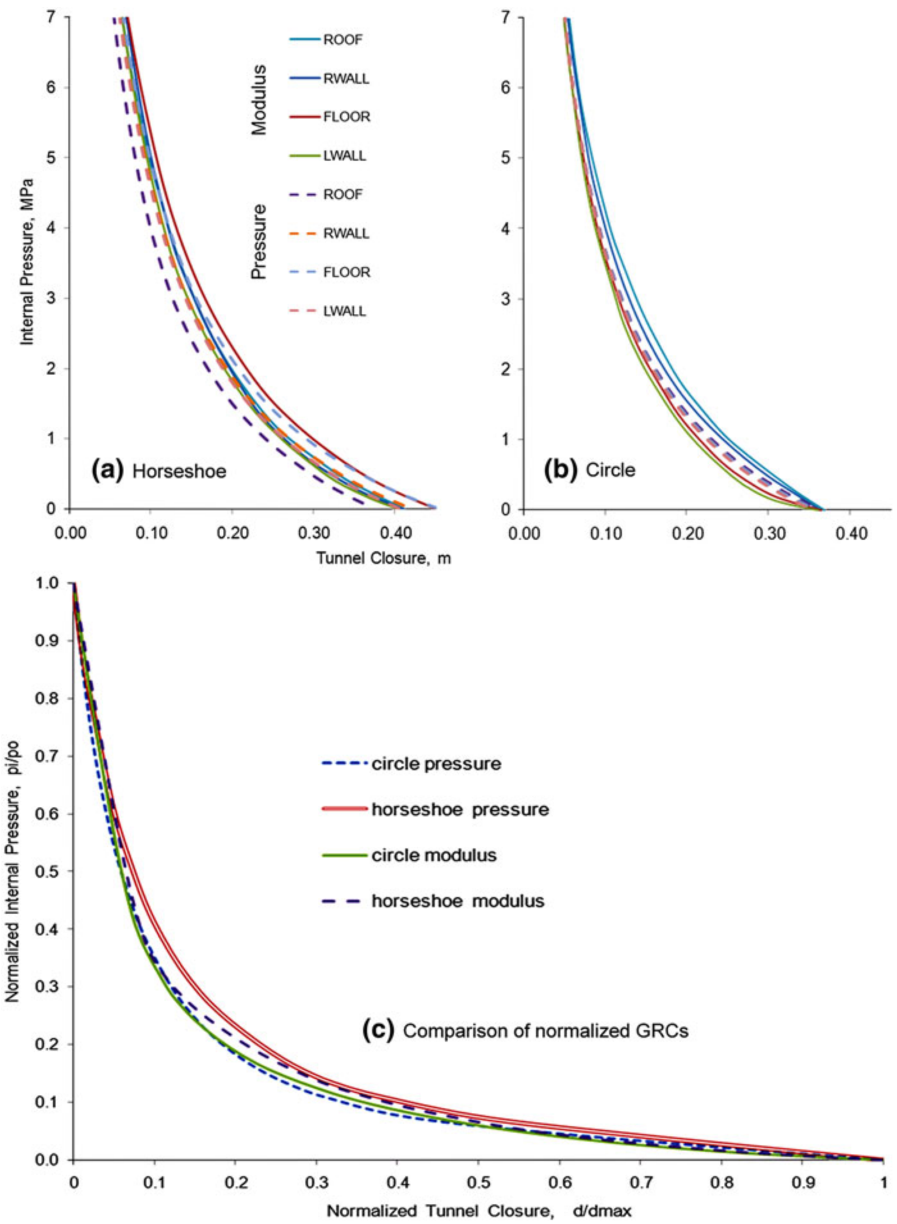

Fig. 12 **a** Comparison of GRC's generated using 2D plain strain analysis, **a** horseshoe tunnel. **b** circular tunnel. Isotropic stress field = 28 MPa, material C from Table 1. "Modulus" refers to the face replacement method while "Pressure" refers to the stress vector method. **c** Comparison of average GRC's generated using 2D plain strain analysis using two methods: "Modulus" refers to the face replacement method while "Pressure" refers to the stress vector method

free boundary conditions with an in situ boundary traction applied (Fig. 6).

The results of the elastic 3D FLAC analysis were compared with the analytical solution for displacements of a circular tunnel (as per Brady and Brown 1993 for example). The results were identical (over a range of elastic moduli) indicating that the boundary conditions and mesh accuracy is acceptable for the 3D models.

Figure 8a illustrates the comparison between the FLAC3D LDP's, normalized with respect to the respective analytical solution for maximum elastic closure, with the 2D solutions based on fixed boundary conditions 6, 12 and 32 radii from the tunnel and with free boundary conditions 12 radii from the tunnel. The latter is exact (coincident with the analytical solution as is the FLAC3D results. Since the kinematic control of a free boundary is more difficult when the tunnel is not circular, fixed 2D boundaries at 16 to 20R are used for the rest of this work.

Finally, it is necessary to compare the normalized LDPs from the FLAC3D analyses with the axisymmetric models used in this paper and compare both to accepted analytical formulations for the longitudinal displacement profile. Figure 8b shows that this normalized profile is coincident with the analytical formulation by Unlu and Gercek (2003). The axisymmetric analysis with a fixed boundary at 30R shows good correlation.

3.2 3D and Axi-symmetrical LDPs

The plastic LDP's for the FLAC 3D models are now compared with the equivalent axisymmetric 2D results

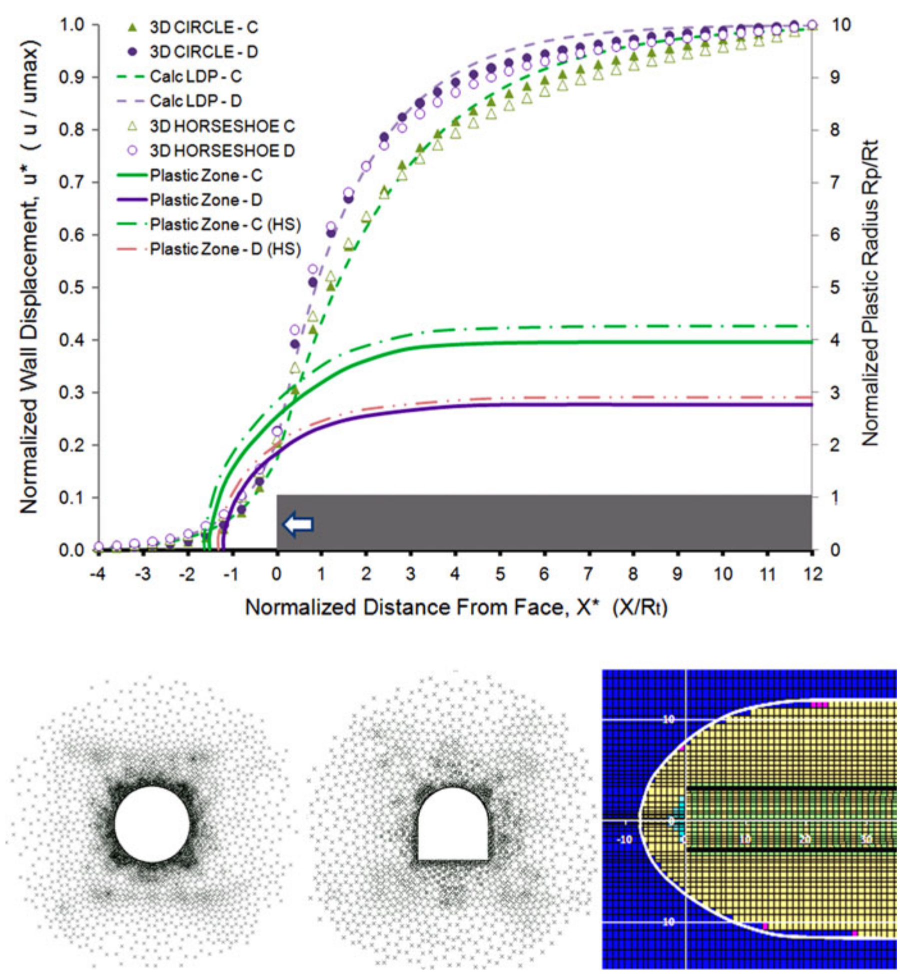

Fig. 13 Comparison of LDPs for *circular and horseshoe tunnels*. Bottom: Plastic zones are shown for the two tunnel shapes and material C. Plastic zone for FLAC 3D (*circle*) is shown in long section

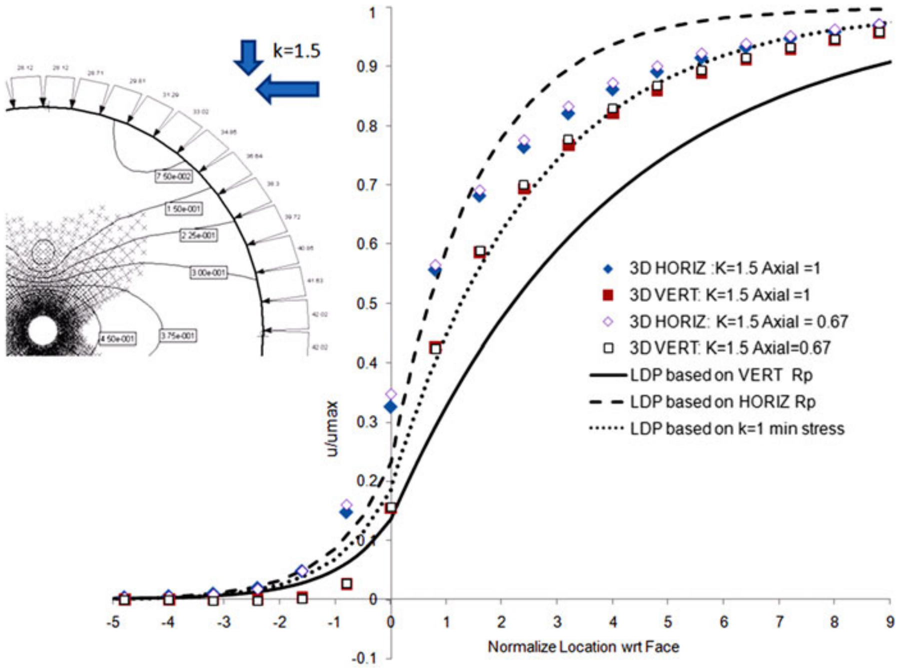

Fig. 14 Comparison of 3D LDPs with derived functions based on Rp. k = 1.5

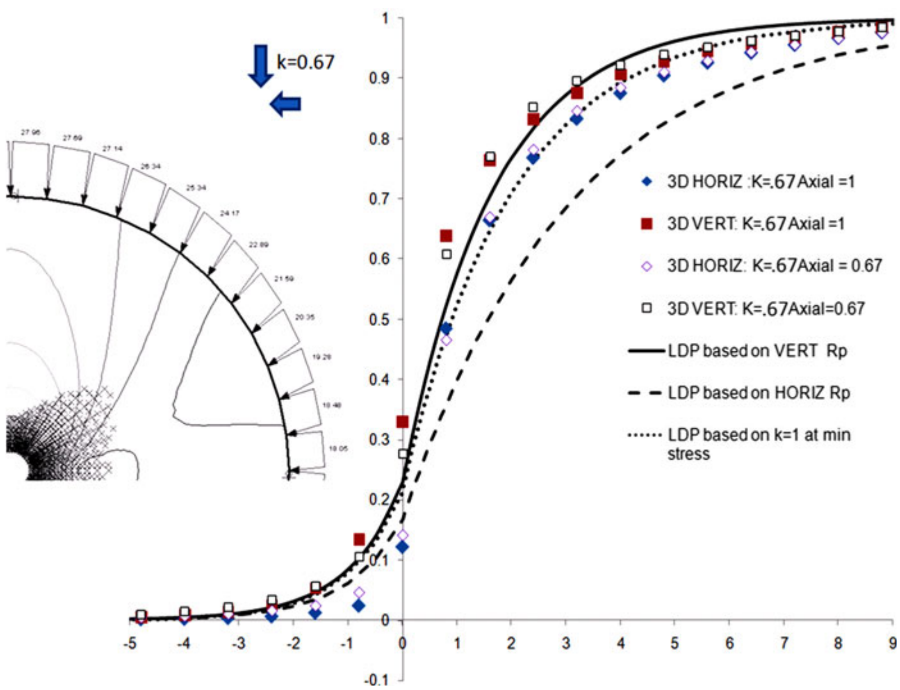

Fig. 15 Comparison of 3D LDPs with derived functions based on Rp. k = 0.67

in Fig. 9 illustrating that they are acceptably coincident. The semi-analytical LDP function proposed by Vlachopoulos and Diederichs (2009) was based on axisymmetric modeling. In Fig. 10, the FLAC3D results are compared with this function. The developing plastic radii from the FLAC3D models are also shown in this figure. The LDP functions are based on the final value of Rp/Rt.

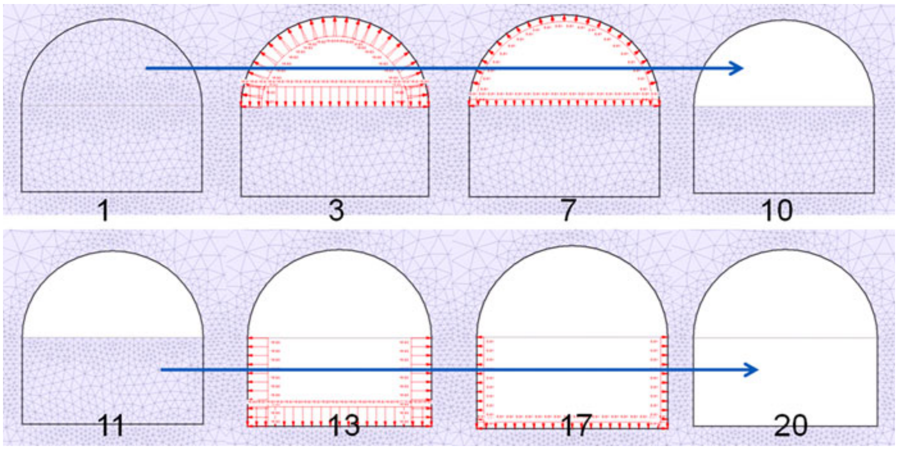

Fig. 16 Selected excavation steps (using internal pressure (stress vector) reduction. Same sequence is used for top heading and bench

3.3 Comparison of 2D Plane Strain Methodologies

An idealized 2D model with a circular tunnel, 6 noded triangular elements arranged in an expanding radial grid with fixed boundaries at 32R from the tunnel, and isotropic stress conditions is initially used to compare the GRCs generated using the stress-vector (pressure) method and the face-replacement (modulus) method described in Sect. 2. In this comparison, 20 steps are used to regenerate the GRC. In order to provide similar load/displacement steps, the "modulus" method is executed first and the internal pressure increments from this analysis are used as input into the "pressure" method. The "internal normal pressure" is queried at the tunnel boundary after each stage. The displacements are given directly. Figure 11 shows that the process is not sensitive to the method used.

Next the same comparison is made using a more practical grid (randomly generated, 3-noded, boundaries at 32R) for both the circular and horseshoe geometries. Results shown in Fig. 12 shows that the "pressure" method is less sensitive to element type (more deviation between roof, floor, wall) in the circular case, and both are subject to deviations caused by non-ideal geometry (in the case of the horseshoe). It is important to keep this inherent error in mind when evaluating the effects of support, stress ratio, sequencing, etc. The average convergence-confinement (GRC) results are compared for two shapes and two methods in Fig. 12c.

4 Limitations of the 2D, LDP Based Simulation of 3D Tunneling

This section will summarize a series of investigations to determine the limitations of 2D FEM modeling to simulate 3D tunnel advance using the LDP approach outlined in Sect. 2.

4.1 Excavation Shape

Figure 13 compares the plastic zone development and the associated LDPs for the circle and horseshoe tunnels under hydrostatic stress. Two rockmass strength/in situ stress ratios are used here. This comparison, combined with Fig. 12 demonstrates that, within the limits of error inherent in the FEM analysis, the LDP plane strain analysis procedure outlined in Sect. 2 is practically valid for non-circular shapes, even if the LDP is based on the correlated LDP functions of Vlachopoulos and Diederichs (2009) for circular tunnels. The validity of this approach is likely reduced as the aspect ratio of the non-circular opening increases.

4.2 In Situ Stress Ratio

The LDP procedures developed for 2D modeling are based on an isotropic stress field ($K = 1$). A brief examination is performed here to determine whether this is a significant limitation for the approach. Figure 14 represents results for material C under a horizontal stress ratio of 1.5 (28 MPa vertical stress).

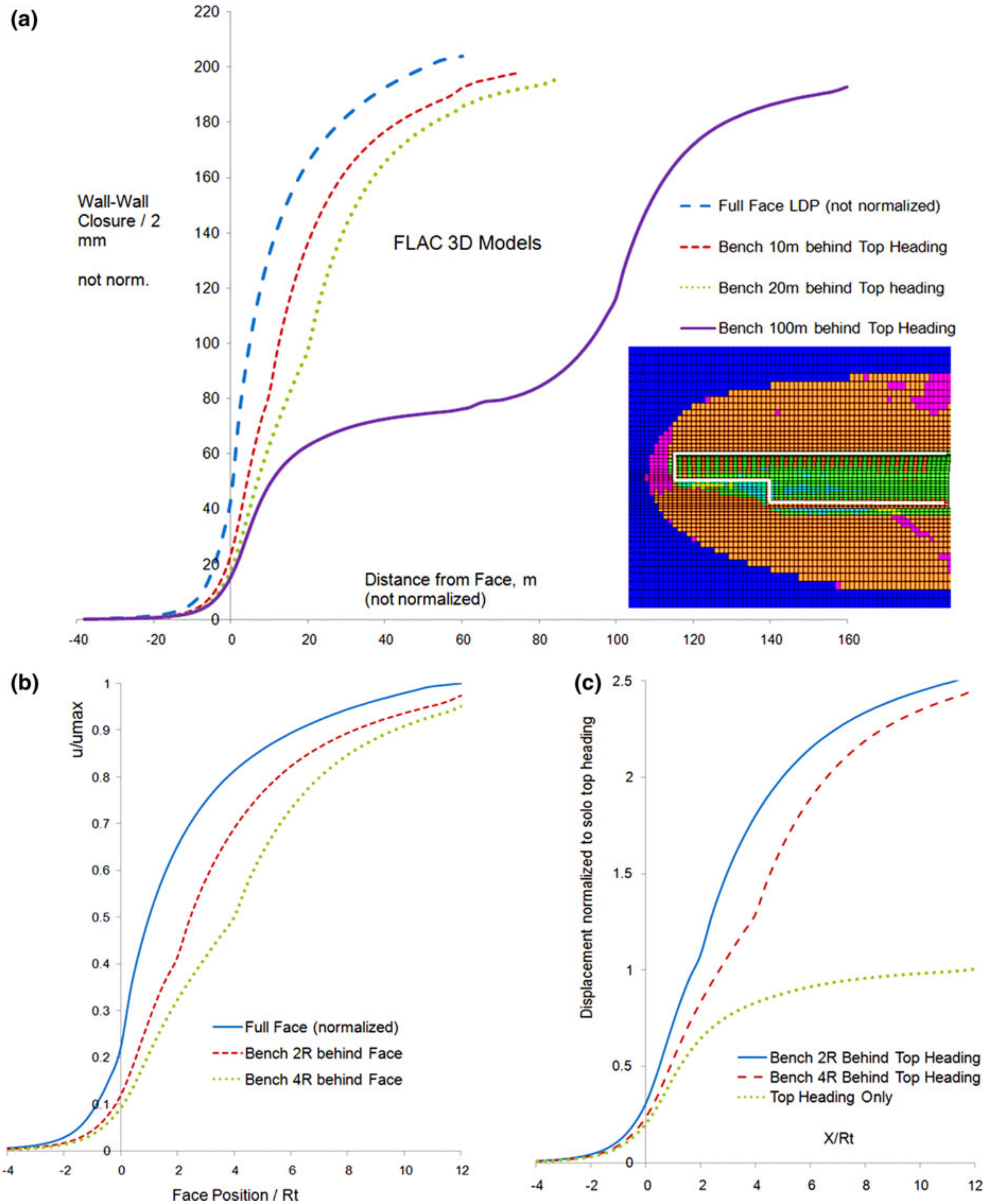

Fig. 17 **a** FLAC3D model LDPs of sequenced excavation (as per *inset*). **b** Normalized LDP's from FLAC3D-normalized to individual maximum displacements and **c** normalized to top heading only

The normalized deformation profiles for the walls for the roof/floor are different. The stress ratio axial to the tunnel has a minimal impact. Using the LDP function derived for isotropic stress (Vlachopoulos and Diederichs (2009)), based on the average volumetric stress in situ), does not work for either horizontal or vertical plastic radius. In this case the field stress vector approach (Sect. 2.4.3) is used as is the face replacement approach (Sect. 2.4.5). The latter is preferred in cases with K significantly different from 1. The average pressure relaxation approach (Sect. (2.4.2)) cannot be used in in the case of anisotropic stress. However, the LDP derived for the case K = 1 does seem to follow the deformation profile for the vertical direction (direction of maximum yield).

This approach would not justify the use of an analytical isotropic stress model or the use of K = 1 in a 2D analysis as it would not consider the considerable moments that would arise in liner support under the anisotropic conditions. It may, however, be practically adequate for approximating the relationship between distance and displacement or load in this case.

Figure 15 illustrates the same comparison with K = 0.67 (same vertical stress). Here again the LDP's are different for different directions even though each LDP is normalized to itself. Interestingly, the LDP for the case of K = 1 best approximates the deformation profile for the horizontal direction (maximum yield). This is consistent with the previous example.

Clearly, these examples show a deviation from the assumptions inherent in this process and point to the need for 3D analysis. For practical purposes, it is evident that using the LDP based on yield for K = 1 can be used in order to calibrate 2D models using the direction of maximum yield and movement. More investigation in this regard is warranted.

4.3 Sequential Excavation

One assumption that is generally accepted in practice is that once a 2D sequenced model is calibrated based on the LDP for a single excavation phase, each subsequent stage can use the same sequence of face replacement or pressure reduction to simulate the 3D advance (of a bench after a top heading for example). This assumption is demonstrated schematically in Fig. 16. Here, an LDP has been established for a single unsupported opening. Through the methodology in Sect. 2.4.3, this LDP is translated to a sequence of

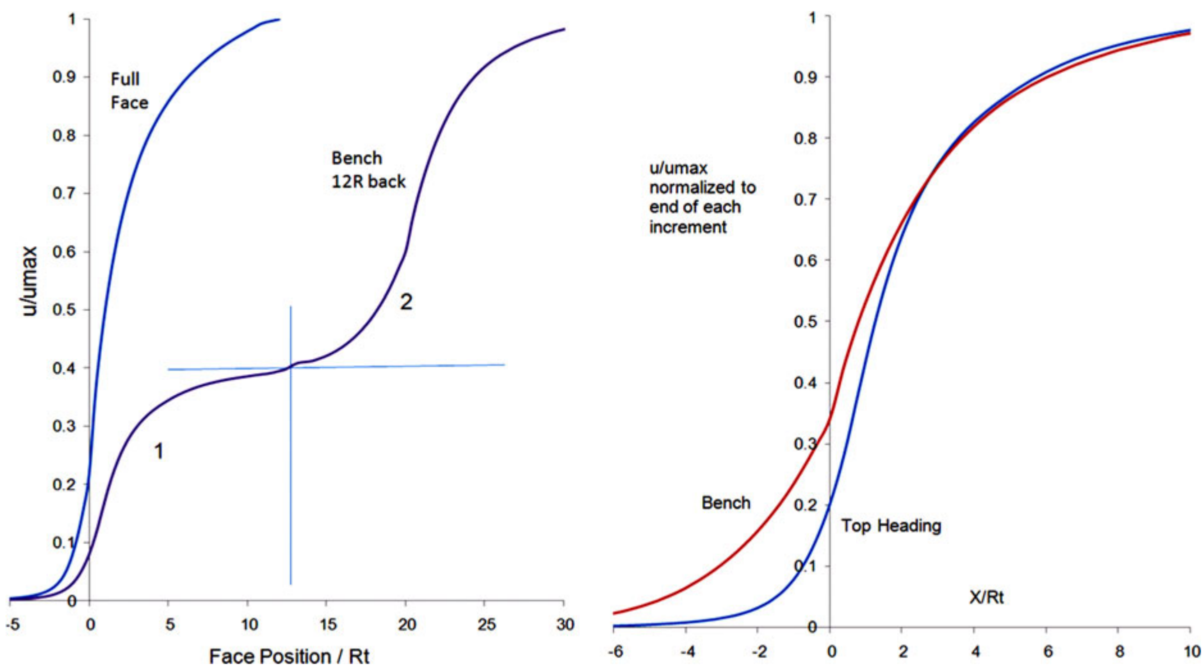

Fig. 18 *Left* full face excavation compared with top heading and bench excavated 20R apart. *Right* normalized LDPs for Segment 1 and 2 from *left image*(isolated heading and bench)

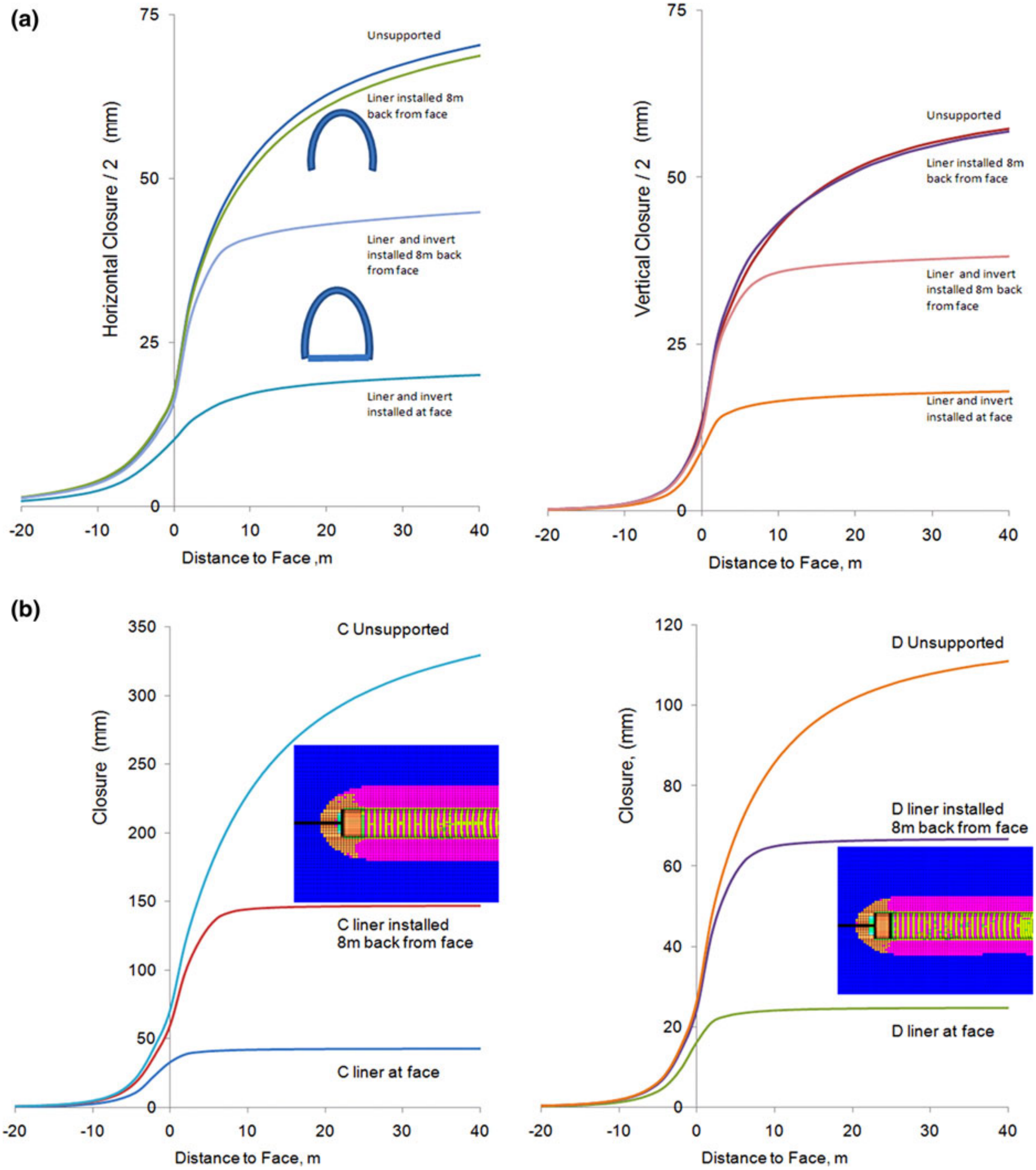

Fig. 19 **a** Unsupported and supported LDP's for *horseshoe* tunnel, generated with FLAC3D. (Material D). **b** Unsupported and supported LDP's for *horseshoe* tunnel, generated with FLAC3D. (Material C and D)

internal pressure reduction increments to stage the excavation in 2D. This is valid as shape is not a major influence. The next step, however, involves using the same sequence of internal pressure reduction to "excavate" the bench. Only selected steps are shown (20 steps in all in this example).

In order to evaluate the validity of this practice, a series of sequenced (top heading and bench)

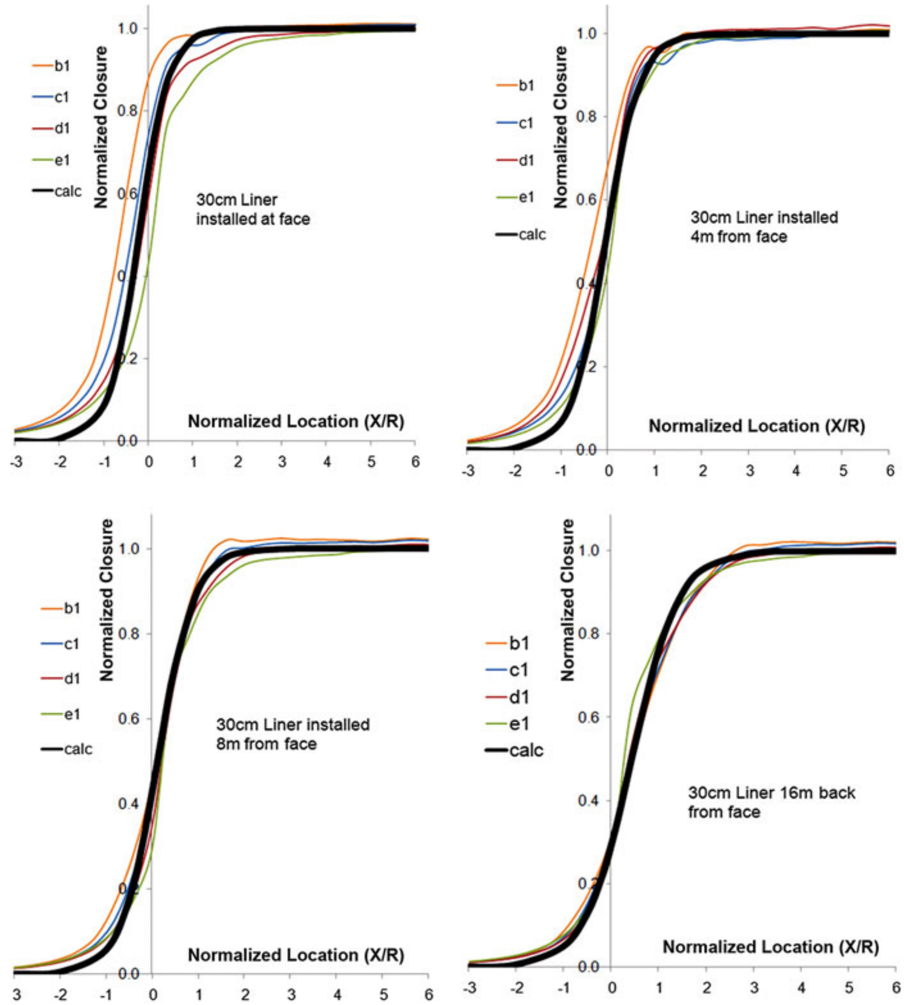

Fig. 20 Supported LDPs (normalized closure vs normalized location) generated through axisymmetric FEM analysis. Materials B,C,D and E were used here. Support installed at different distances from the face. Best fit Sigmoid function (Equation 1) overlain

excavation simulations were analyzed using FLAC3D as shown in Fig. 17.

Figure 17 illustrates a potential difficulty with the standard 2D approach. The normalized LDPs for different timings of bench excavation differ considerably from the single excavation (in terms of percentage of displacement at the passing of the bench). The convergence confinement approach (analytical or in 2D plane strain) assumes that the excavation stages are independent. This is not the case here as the approaching bench ‘softens’ the ongoing response of the initial top heading excavation (as the tunnel face moves on). Figure 18, on the other hand, shows that for practical purposes, the same sequence may be used for a second excavation stage provided the distance between the stages is sufficient for them to act independently.

5 Support Modelling

The primary purpose of convergence-confinement analysis using LDPs is to properly locate support installations within the deforming tunnel to optimize the liner resistance without exceeding the liner strength. The methodology is the same (using Sect. 2) for plane strain analysis. This section will examine the installation of a 30 cm concrete lining with steel sets at 1 m spacing along the tunnel. This support will be installed at different steps in the model.

5.1 Excavation Shape

Figure 19 illustrates LDPs generated trough FLAC3D analysis for different excavation shape, liner configurations and support sequencing. In all cases for this illustrative analysis, the liner composition is assumed to be a 30 cm thick shotcrete layer with a 16 cm steel H-channel ($I = 2.2e\text{-}005\ m^4$) centred with the liner and spaced 1 m along tunnel (shotcrete is continuous). The shotcrete modulus is 30GPa and the liner rock interface is assumed to be fully bonded. It can be seen from these set of analyses, that the LDPs for the supported and unsupported tunnels are similar up to a point just before installation as long as the liner is several radii from the face.

When the liner is less than 3 radii from the face, the error incurred in using the unsupported LDP to locate the installation point for the liner may become unacceptable. In order to address this problem, at least for the case of isotropic stresses, a series of axisymmetric runs with staged liners was analyzed. The normalized LDP's are shown in Fig. 20. The LDP's are consistently sigmoidal in nature and relatively insensitive to material type when normalized.

A version of a Sigmoid function was developed here to provide a best fit LDP as a function of face distance and support installation position:

$$\frac{u}{u_{\max}} = \frac{1}{1 + e^{0.6\left(1-0.1\frac{S}{R}\right)\left(\frac{S}{R}-5\frac{X}{R}-1\right)}} \qquad (1)$$

where X is the distance from the face, R is the tunnel radius and S is the distance between the face and the support. This phenomenological adjustment is not valid as S/R approaches and exceeds 10. The function is shown against the model data in Fig. 20. In this case, u_{max} is the maximum supported displacement and is of course a function of the selected support location. As such this function must be used to iteratively position support in a 2D model to achieve the correct ration between face displacement and final supported displacement.

6 Conclusions

The conventional approach of 2D tunnel analysis, calibrating excavation stages with an LDP derived from simple 3D calculations based on an unsupported circular tunnel in isotropic stresses, has been examined in detail in this paper. The results indicate that for tunnel analyses with sequenced support, excavation steps or non-isotropic stresses, 3D analyses are generally recommended for engineering calculations beyond the preliminary phase of design. Where 2D analyses are used then the following caveats should be observed:

- Boundary conditions are an important component of the analysis of squeezing ground problems. Fixed boundaries should be a minimum of 12 radii from the tunnel or at least 3 plastic radii away from the plastic zone.
- For simple tunnel geometries, the 2D LDP and GRC are not sensitive to the choice of face replacement or pressure reduction technique but are sensitive to the step size (face too soft or pressure increment too great).
- Tunnel shape is an important factor for the application of 2D staged modeling although results can be practically acceptable provided the aspect ratio of tunnel geometry is not extreme.
- Non-isotropic stresses render the standard LDP approach inaccurate. For moderate values of stress ratio, K, some assumptions and adjustments can be made to make the approach practically viable.
- Sequenced excavation such as top heading and bench excavation poses a problem for the LDP approach unless the second excavation stage is distant from the first.
- A revised LDP (such as that proposed by Cantieni and Anagnostou (2009) is required for stiff liners installed within 2 to 6 radii of the face. For installations closer than 2 radii, 3D analysis may be required.
- It is critical to correctly locate the installation step within a staged 2D modeling sequence to prevent overloading or excess deformations.

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